12.2 Surface Area of Prisms and Cylinders

Before You found areas of polygons.

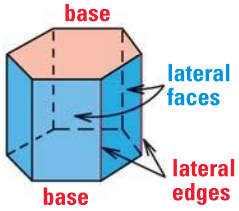
Now You will find the surface areas of prisms and cylinders.

Why? So you can find the surface area of a drum, as in Ex. 22.

Key Vocabulary

- **prism**
  lateral faces, lateral edges
- **surface area**
- **lateral area**
- **net**
- **right prism**
- **oblique prism**
- **cylinder**
- **right cylinder**

A **prism** is a polyhedron with two congruent faces, called *bases*, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are **lateral edges**. Prisms are classified by the shapes of their bases.



The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

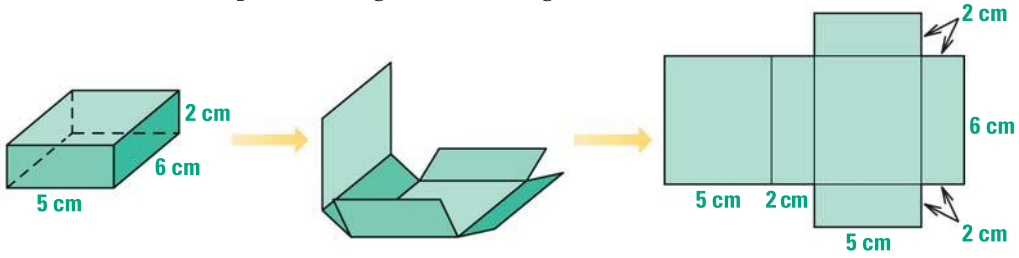
Imagine that you cut some edges of a polyhedron and unfold it. The two-dimensional representation of the faces is called a **net**. As you saw in the Activity on page 802, the surface area of a prism is equal to the area of its net.

EXAMPLE 1 Use the net of a prism

Find the surface area of a rectangular prism with height 2 centimeters, length 5 centimeters, and width 6 centimeters.

Solution

STEP 1 **Sketch** the prism. Imagine unfolding it to make a net.



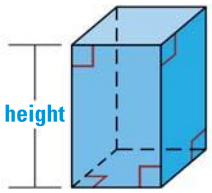
STEP 2 **Find** the areas of the rectangles that form the faces of the prism.

Congruent faces	Dimensions	Area of each face
Left and right faces	6 cm by 2 cm	$6 \cdot 2 = 12$ cm^2
Front and back faces	5 cm by 2 cm	$5 \cdot 2 = 10$ cm^2
Top and bottom faces	6 cm by 5 cm	$6 \cdot 5 = 30$ cm^2

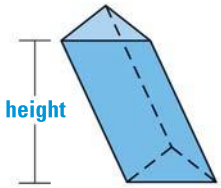
STEP 3 **Add** the areas of all the faces to find the surface area.

▶ The surface area of the prism is $S = 2(12) + 2(10) + 2(30) = 104$ cm^2.

RIGHT PRISMS The height of a prism is the perpendicular distance between its bases. In a **right prism**, each lateral edge is perpendicular to both bases. A prism with lateral edges that are not perpendicular to the bases is an **oblique prism**.



Right rectangular prism



Oblique triangular prism

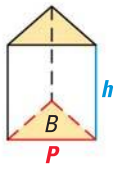
THEOREM *For Your Notebook*

THEOREM 12.2 Surface Area of a Right Prism

The surface area S of a right prism is

$$S = 2B + Ph = aP + Ph,$$

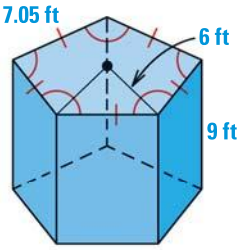
where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height.



$S = 2B + Ph = aP + Ph$

EXAMPLE 2 Find the surface area of a right prism

Find the surface area of the right pentagonal prism.



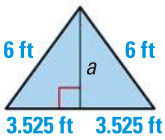
Solution

STEP 1 **Find** the perimeter and area of a base of the prism.

Each base is a regular pentagon.

Perimeter $P = 5(7.05) = 35.25$

Apothem $a = \sqrt{6^2 - 3.525^2} \approx 4.86$



REVIEW APOTHEM
For help with finding the apothem, see p. 762.

STEP 2 **Use** the formula for the surface area that uses the apothem.

$S = aP + Ph$ **Surface area of a right prism**

$\approx (4.86)(35.25) + (35.25)(9)$ **Substitute known values.**

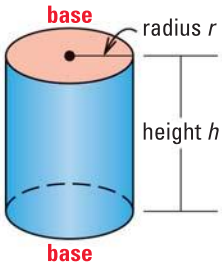
≈ 488.57 **Simplify.**

▶ The surface area of the right pentagonal prism is about 488.57 square feet.

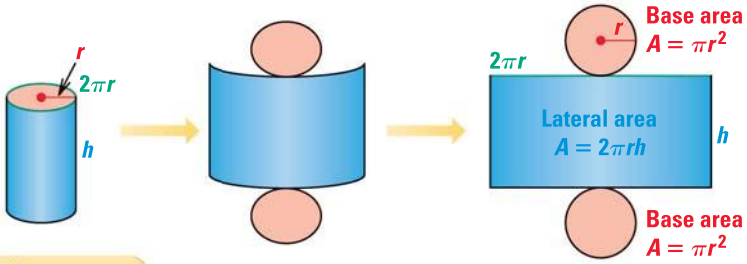
GUIDED PRACTICE for Examples 1 and 2

1. Draw a net of a triangular prism.
2. Find the surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches using (a) a net and (b) the formula for the surface area of a right prism.

CYLINDERS A **cylinder** is a solid with congruent circular bases that lie in parallel planes. The height of a cylinder is the perpendicular distance between its bases. The radius of a base is the *radius* of the cylinder. In a **right cylinder**, the segment joining the centers of the bases is perpendicular to the bases.



The lateral area of a cylinder is the area of its curved surface. It is equal to the product of the circumference and the height, or $2\pi rh$. The surface area of a cylinder is equal to the sum of the lateral area and the areas of the two bases.



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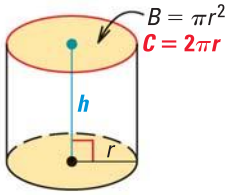
THEOREM *For Your Notebook*

THEOREM 12.3 Surface Area of a Right Cylinder

The surface area S of a right cylinder is

$$S = 2B + Ch = 2\pi r^2 + 2\pi rh,$$

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.



$S = 2B + Ch = 2\pi r^2 + 2\pi rh$

EXAMPLE 3 Find the surface area of a cylinder

COMPACT DISCS You are wrapping a stack of 20 compact discs using a shrink wrap. Each disc is cylindrical with height 1.2 millimeters and radius 60 millimeters. What is the minimum amount of shrink wrap needed to cover the stack of 20 discs?

Solution

The 20 discs are stacked, so the height of the stack will be $20(1.2) = 24$ mm. The radius is 60 millimeters. The minimum amount of shrink wrap needed will be equal to the surface area of the stack of discs.

$S = 2\pi r^2 + 2\pi rh$ — Surface area of a cylinder

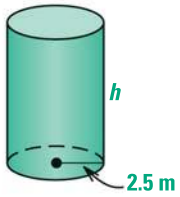
$= 2\pi(60)^2 + 2\pi(60)(24)$ — Substitute known values.

$\approx 31{,}667$ — Use a calculator.

▶ You will need at least 31,667 square millimeters, or about 317 square centimeters of shrink wrap.

EXAMPLE 4 Find the height of a cylinder

Find the height of the right cylinder shown, which has a surface area of 157.08 square meters.



Solution

Substitute known values in the formula for the surface area of a right cylinder and solve for the height h.

$$S = 2\pi r^2 + 2\pi rh \qquad \text{Surface area of a cylinder}$$

$$157.08 = 2\pi(2.5)^2 + 2\pi(2.5)h \qquad \text{Substitute known values.}$$

$$157.08 = 12.5\pi + 5\pi h \qquad \text{Simplify.}$$

$$157.08 - 12.5\pi = 5\pi h \qquad \text{Subtract } 12.5\pi \text{ from each side.}$$

$$117.81 \approx 5\pi h \qquad \text{Simplify. Use a calculator.}$$

$$7.5 \approx h \qquad \text{Divide each side by } 5\pi.$$

▶ The height of the cylinder is about 7.5 meters.

GUIDED PRACTICE for Examples 3 and 4

3. Find the surface area of a right cylinder with height 18 centimeters and radius 10 centimeters. Round your answer to two decimal places.

4. Find the radius of a right cylinder with height 5 feet and surface area 208π square feet.

12.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 24, 25, and 26

SKILL PRACTICE

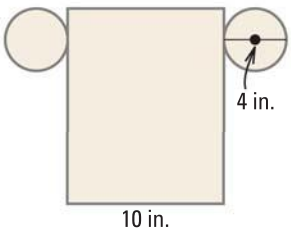
1. VOCABULARY Sketch a triangular prism. Identify its *bases*, *lateral faces*, and *lateral edges*.

2. ★ WRITING *Explain* how the formula $S = 2B + Ph$ applies to finding the surface area of both a right prism and a right cylinder.

EXAMPLE 1 on p. 803 for Exs. 3–5

USING NETS **Find the surface area of the solid formed by the net. Round your answer to two decimal places.**

3.



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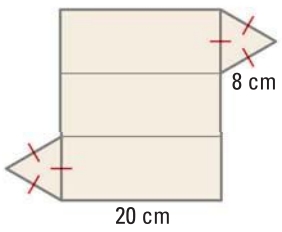

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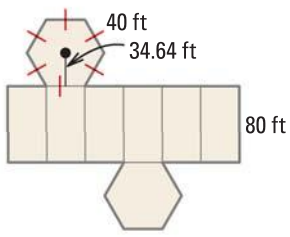
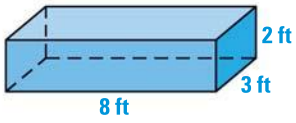


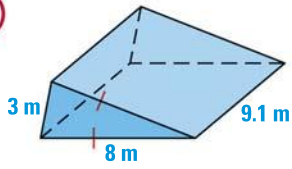
EXAMPLE 2 on p. 804 for Exs. 6–8

SURFACE AREA OF A PRISM **Find the surface area of the right prism. Round your answer to two decimal places.**

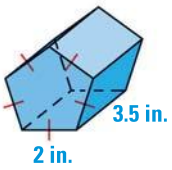
6.



7.



8.



EXAMPLE 3 on p. 805 for Exs. 9–12

SURFACE AREA OF A CYLINDER **Find the surface area of the right cylinder using the given radius r and height h. Round your answer to two decimal places.**

9.

$r = 0.8$ in.
$h = 2$ in.

10.

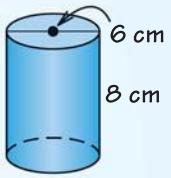
$r = 12$ mm
$h = 40$ mm

11.

$r = 8$ in.
$h = 8$ in.

12. ERROR ANALYSIS *Describe* and correct the error in finding the surface area of the right cylinder.

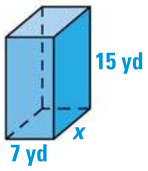
$$S = 2\pi(6^2) + 2\pi(6)(8)$$
$$= 2\pi(36) + 2\pi(48)$$
$$= 168\pi$$
$$\approx 528 \text{ cm}^2$$



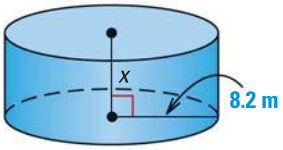
EXAMPLE 4 on p. 806 for Exs. 13–15

ALGEBRA **Solve for x given the surface area S of the right prism or right cylinder. Round your answer to two decimal places.**

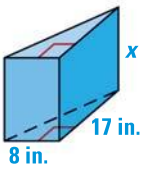
13. $S = 606 \text{ yd}^2$



14. $S = 1097 \text{ m}^2$



15. $S = 616 \text{ in.}^2$



16. SURFACE AREA OF A PRISM A triangular prism with a right triangular base has leg length 9 units and hypotenuse length 15 units. The height of the prism is 8 units. Sketch the prism and find its surface area.

17. ★ MULTIPLE CHOICE The length of each side of a cube is multiplied by 3. What is the change in the surface area of the cube?

Ⓐ The surface area is 3 times the original surface area.

Ⓑ The surface area is 6 times the original surface area.

Ⓒ The surface area is 9 times the original surface area.

Ⓓ The surface area is 27 times the original surface area.

18. SURFACE AREA OF A CYLINDER The radius and height of a right cylinder are each divided by $\sqrt{5}$. What is the change in surface area of the cylinder?

19. SURFACE AREA OF A PRISM Find the surface area of a right hexagonal prism with all edges measuring 10 inches.

20. HEIGHT OF A CYLINDER Find the height of a cylinder with a surface area of 108π square meters. The radius of the cylinder is twice the height.

21. CHALLENGE The *diagonal* of a cube is a segment whose endpoints are vertices that are not on the same face. Find the surface area of a cube with diagonal length 8 units. Round your answer to two decimal places.

PROBLEM SOLVING

EXAMPLE 3 on p. 805 for Ex. 22

22. BASS DRUM A bass drum has a diameter of 20 inches and a depth of 8 inches. Find the surface area of the drum.

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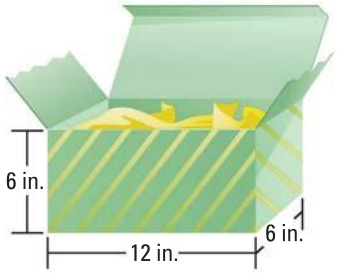
23. GIFT BOX An open gift box is shown at the right. When the gift box is closed, it has a length of 12 inches, a width of 6 inches, and a height of 6 inches.



a. What is the minimum amount of wrapping paper needed to cover the closed gift box?

b. Why is the area of the net of the box larger than the amount of paper found in part (a)?

c. When wrapping the box, why would you want more paper than the amount found in part (a)?

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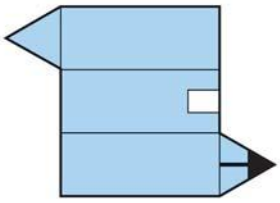
24. ★ EXTENDED RESPONSE A right cylinder has a radius of 4 feet and height of 10 feet.

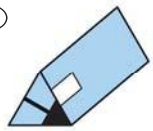
a. Find the surface area of the cylinder.

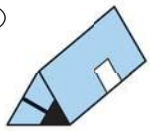
b. Suppose you can either *double the radius* or *double the height*. Which do you think will create a greater surface area?

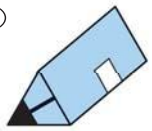
c. Check your answer in part (b) by calculating the new surface areas.

25. ★ MULTIPLE CHOICE Which three-dimensional figure does the net represent?



Ⓐ 

Ⓑ 

Ⓒ 

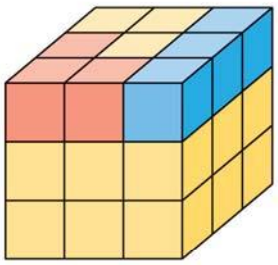
Ⓓ 

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

26. ★ **SHORT RESPONSE** A company makes two types of recycling bins. One type is a right rectangular prism with length 14 inches, width 12 inches, and height 36 inches. The other type is a right cylinder with radius 6 inches and height 36 inches. Both types of bins are missing a base, so the bins have one open end. Which recycle bin requires more material to make? *Explain*.

27. **MULTI-STEP PROBLEM** Consider a cube that is built using 27 unit cubes as shown at the right.

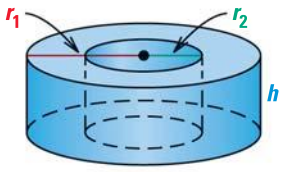


a. Find the surface area of the solid formed when the red unit cubes are removed from the solid shown.

b. Find the surface area of the solid formed when the blue unit cubes are removed from the solid shown.

c. Why are your answers different in parts (a) and (b)? *Explain*.

28. **SURFACE AREA OF A RING** The ring shown is a right cylinder of radius r_1 with a cylindrical hole of r_2. The ring has height h.

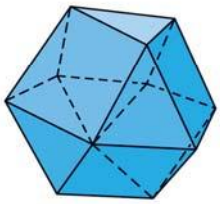


a. Find the surface area of the ring if r_1 is 12 meters, r_2 is 6 meters, and h is 8 meters. Round your answer to two decimal places.

b. Write a formula that can be used to find the surface area S of any cylindrical ring where $0 < r_2 < r_1$.

29. **DRAWING SOLIDS** A cube with edges 1 foot long has a cylindrical hole with diameter 4 inches drilled through one of its faces. The hole is drilled perpendicular to the face and goes completely through to the other side. Draw the figure and find its surface area.

30. **CHALLENGE** A cuboctahedron has 6 square faces and 8 equilateral triangle faces, as shown. A cuboctahedron can be made by slicing off the corners of a cube.



a. Sketch a net for the cuboctahedron.

b. Each edge of a cuboctahedron has a length of 5 millimeters. Find its surface area.

MIXED REVIEW

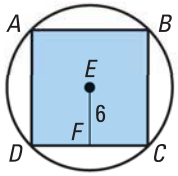
The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(p. 507)*

31. 1260° **32.** 1080° **33.** 720° **34.** 1800°

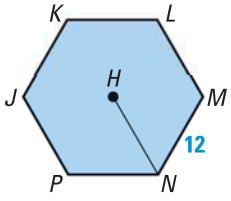
PREVIEW
Prepare for Lesson 12.3 in Exs. 35–37.

Find the area of the regular polygon. *(p. 762)*

35.



36.



37.